Worksheet week 2, October 2nd, 2017, MATH 1510 D

1. $C(-6,3)$, $D(3,2)$, $E(-2,1)$

$$d_{CE} = \left((x_2-x_1)^2 + (y_2-y_1)^2\right)^{1/2} = \left((-2-(-6))^2 + (1-3)^2\right)^{\frac{1}{2}} = \left((4)^2 + (-2)^2\right)^{\frac{1}{2}}$$
$$= (16+4)^{1/2} = \sqrt{20}$$

$$d_{DE} = \left((-2-3)^2 + (1-2)^2\right)^{\frac{1}{2}} = \left((-5)^2 + (-1)^2\right)^{\frac{1}{2}} = (25+1)^{\frac{1}{2}} = \sqrt{26}$$

Thus, point C is closer to E, as the distance between C and E is shorter than between D and E.

2. Point on y-axis $\Rightarrow$ x coordinate of this point is 0. Let P be the name of the point we are looking for. Therefore, $P = (0, y_3)$ is what we want.

Points: $A = (4,-4)$, $B = (2,2)$.

Equidistant = same distance between A and B. That means, $d_{PA} = d_{PB}$, and we solve for the y-coordinate of P.

So, $d_{PA} = \left((0-4)^2 + (y_3-(-4))^2\right)^{\frac{1}{2}} = \left((-4)^2 + (y_3+4)^2\right)^{\frac{1}{2}} = \left((y_3+4)^2 + 16\right)^{\frac{1}{2}}$

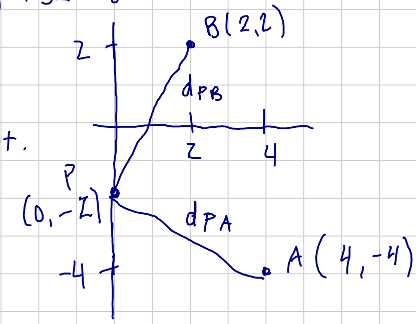
Also, $d_{PB} = \left((0-2)^2 + (y_3-2)^2\right)^{\frac{1}{2}} = \left((-2)^2 + (y_3-2)^2\right)^{\frac{1}{2}} = \left((y_3-2)^2 + 4\right)^{\frac{1}{2}}$.

Equidistant means $d_{PA} = d_{PB} \Rightarrow \left((y_3+4)^2 + 16\right)^{\frac{1}{2}} = \left((y_3-2)^2 + 4\right)^{\frac{1}{2}}$.

We need to now solve for $y_3$, and then we will have found the coordinates of P.

$$\left((y_3+4)^2+16\right)^{1/2} = \left((y_3-2)^2+4\right)^{1/2}$$
$$\left[\left((y_3+4)^2+16\right)^{\frac{1}{2}}\right]^2 = \left[\left((y_3-2)^2+4\right)^{\frac{1}{2}}\right]^2 \Rightarrow (y_3+4)^2 + 16 = (y_3-2)^2 + 4$$
$$y_3^2 + 2(y_3)(4) + 16 + 16 = y_3^2 + 2(y_3)(-2) + 4 + 4$$
$$y_3^2 + 8y_3 + 32 = y_3^2 - 4y_3 + 8 \Rightarrow y_3^2 - y_3^2 + 8y_3 - (-4y_3) + 32 - 8 = 0$$
$$12y_3 + 24 = 0 \Rightarrow \frac{12y_3}{12} = \frac{-24}{12} \Rightarrow y_3 = -2.$$

Thus, $P = (0,-2)$ is the point on the y-axis that is equidistant.



3. General form of circle $(x-x_0)^2+(y-y_0)^2=r^2$

where $(x_0, y_0)$ is the centre of the circle, and $r$ is the radius.

So, Centre $(2,-5) \Rightarrow x_0 = 2,\ y_0 = -5$, and radius $2 \Rightarrow r = 2$.

The equation is $(x-2)^2+(y-(-5))^2=(2)^2$

$$(x-2)^2+(y+5)^2=4.$$

4. Centre $(-1,5)$, and passing through $(-9,-7)$.

The point $(-9,-7)$ must sit on the circumference of the circle, so we can use it to determine the radius of the circle.

By the general form, $(x+1)^2+(y-5)^2=r^2$,

The point $(-9,-7)$ must be distance = radius away from the centre. Therefore, if we can figure out the distance between the point and the centre of the circle, we know the radius.

Let $P=(-9,-7)$, and centre $C(-1,5)$.

$$d_{PC} = \left((-9-(-1))^2+(-7-5)^2\right)^{1/2} = \left((-8)^2+(-12)^2\right)^{1/2} = (64+144)^{1/2} = \sqrt{208}$$

Therefore $r=\sqrt{208}$. The equation is then, $(x+1)^2+(y-5)^2=(\sqrt{208})^2$

$$(x+1)^2+(y-5)^2=208.$$

5. Endpoints of diameter $P(-2,2)$, $Q(6,8)$.

Diameter = 2 radius. So, the diameter is $d_{PQ}$, distance between the two endpoints.

$$d_{PQ} = \text{diameter} = \left((-2-6)^2+(2-8)^2\right)^{1/2} = \left((-8)^2+(-6)^2\right)^{1/2} = (64+36)^{1/2} = \sqrt{100} = 10.$$

So, $2r = d_{PQ} = 10 \Rightarrow r = \frac{10}{2} = 5.$

The centre of the circle is the midpoint of the diameter. Centre $C(x_1, y_1)$.

$$x_1 = \frac{1}{2}(-2+6) = \frac{1}{2}(4) = 2. \quad y_1 = \frac{1}{2}(2+8) = \frac{1}{2}(10) = 5.$$

So, the equation of the circle is $(x-2)^2+(y-5)^2=(5)^2 \Rightarrow (x-2)^2+(y-5)^2=25.$

The radius is 5 and the centre is $(2,5)$.

6. The Standard form of a circle is $(x-x_0)^2+(y-y_0)^2=r^2$, where $(x_0, y_0)$ is the centre.

$x^2+y^2+4x-8y+19=0$. We will have to complete the square in $x$ and $y$.

Let's do $x$ first: $x^2+4x+y^2-8y+19=0$.
Coefficient of $x$ term: 4.
So, $\left(\frac{4}{2}\right)^2 = 2^2 = 4$. Therefore, we add in 4 and subtract 4 (what we add we must take away!)

$$\underbrace{x^2+4x+4}-4+y^2-8y+19=0$$

Now we can factor! We already know how to factor, because it is just the term we had above: $\frac{4}{2}=2$.

So, $x^2+4x+4=(x+2)^2$ (Check this yourself!)

$\Rightarrow (x+2)^2-4+y^2-8y+19=0$. Let's now complete the square for $y$: $-8y \Rightarrow \left(\frac{-8}{2}\right)^2=(-4)^2=16$.

This is the term we get in the factor

So we need to add and subtract 16 to complete the square properly.

$$(x+2)^2-4+\underbrace{y^2-8y+16}_{\text{Now we factor this!}}-16+19=0$$

$y^2-8y+16=(y-4)^2$ Same thing!

$\Rightarrow (x+2)^2+(y-4)^2-4-16+19=0$

$(x+2)^2+(y-4)^2-1=0 \Rightarrow (x+2)^2+(y-4)^2=1$. This is the standard form as is indeed a circle!

Centre: $C(-2, 4)$; radius $= 1$.

7. $2x^2+2y^2-7x=0$.

We will complete the square again! $2x^2-7x+2y^2=0 \Rightarrow x^2-\frac{7}{2}x+y^2=0$.

In $x$: $\left(-\frac{7}{2}\times\frac{1}{2}\right)^2=\left(-\frac{7}{4}\right)^2=\frac{49}{16}$.

$$\Rightarrow \underbrace{x^2-\frac{7}{2}x+\frac{49}{16}}_{\text{factor!}}-\frac{49}{16}+y^2=0 \Rightarrow \left(x-\frac{7}{4}\right)^2+y^2-\frac{49}{16}=0$$

$\Rightarrow \left(x-\frac{7}{4}\right)^2+y^2=\frac{49}{16}$, which is the standard form of a circle.

$$r^2=\frac{49}{16} \Rightarrow (r^2)^{\frac{1}{2}}=r=\left(\frac{49}{16}\right)^{\frac{1}{2}}=\sqrt{\frac{49}{16}}=\frac{\sqrt{49}}{\sqrt{16}}=\frac{7}{4}.$$

So, Centre $C=(7/4, 0)$, radius $r=\frac{7}{4}$.

8. $\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1.$

a) Reading off the graph; Closest : 2 Mm (on the left of the moon)
furthest : 8 Mm (on the right of the moon)

b) y-coordinate = 2. So, we have two points where $y=2$. Let's call them $(x_1, 2)$ and $(x_2, 2)$.
Now we substitute each point in and solve for $x_1$ and $x_2$

$$\frac{(x-3)^2}{25} + \frac{(2)^2}{16} = 1 \Rightarrow \frac{(x-3)^2}{25} + \frac{4}{16} = 1 \Rightarrow \frac{(x-3)^2}{16} + \frac{1}{4} = 1$$

$$\Rightarrow \frac{(x-3)^2}{16} = \frac{3}{4} \Rightarrow (x-3)^2 = \left(\frac{3}{4}\right)(16) \Rightarrow (x-3)^2 = 12 \Rightarrow x-3 = \pm\sqrt{12}$$

$$\Rightarrow x = 3 \pm \sqrt{12} = 3 \pm \sqrt{4\times3} = 3 \pm 2\sqrt{3}.$$

So, we have $x_1 = 3+2\sqrt{3}$ and $x_2 = 3-2\sqrt{3}$ (we take the + for one coordinate and the − for the other)

Therefore, the points are $(3+2\sqrt{3}, 2)$ and $(3-2\sqrt{3}, 2)$.

$3-2\sqrt{3}$ is smaller than $3+2\sqrt{3}$.

Distances to the centre of the moon:

1) Smaller x-coordinate: $(3-2\sqrt{3}, 2)$

$$d_1 = \sqrt{(3-2\sqrt{3}-0)^2 + (2-0)^2} = \sqrt{(3-2\sqrt{3})^2 + 4} = \sqrt{9 + 2(3)(-2\sqrt{3}) + (-2\sqrt{3})^2 + 4}$$

$$= \sqrt{9 - 12\sqrt{3} + (4)(3) + 4} = \sqrt{25 - 12\sqrt{3}} \approx 2.05 \text{ Mm}$$

2) Larger x-coordinate: $(3+2\sqrt{3}, 2)$

$$d_2 = \sqrt{(3+2\sqrt{3})^2 + (2-0)^2} = \sqrt{9 + 2(3)(2\sqrt{3}) + 4(3) + 4} = \sqrt{25 + 12\sqrt{3}} \approx 6.46 \text{ Mm}.$$