Week 5, October 23, 2017

1. $h(x) = \frac{x^2+4}{5}$, $h(2) = \frac{2^2+4}{5} = \frac{8}{5}$; $h(-2) = \frac{(-2)^2+4}{5} = \frac{8}{5}$; $h(a) = \frac{a^2+4}{5}$; $h(-x) = \frac{(-x)^2+4}{5} = \frac{x^2+4}{5}$

$h(a-2) = \frac{(a-2)^2+4}{5} = \frac{a^2-4a+4+4}{5} = \frac{a^2-4a+8}{5}$; $h(\sqrt{x}) = \frac{(\sqrt{x})^2+4}{5} = \frac{x+4}{5}$.

2. $g(x) = \frac{6-x}{6+x}$. $g(2) = \frac{6-2}{6+2} = \frac{4}{8} = \frac{1}{2}$; $g(-6) = \frac{6-(-6)}{6+(-6)} = \frac{12}{0}$, undefined.

$g(\frac{1}{2}) = \frac{6-\frac{1}{2}}{6+\frac{1}{2}} = \frac{12/2 - 1/2}{12/2 + 1/2} = \frac{11/2}{13/2} = \frac{11}{2} \times 1/\frac{13}{2} = \left(\frac{11}{2}\right)\left(\frac{2}{13}\right) = \frac{11}{13}$; $g(a) = \frac{6-a}{6+a}$.

$g(a-6) = \frac{6-(a-6)}{6+(a-6)} = \frac{12-a}{a}$; $g(x^2-6) = \frac{6-(x^2-6)}{6+(x^2-6)} = \frac{12-x^2}{x^2}$.

3. $f(x) = \begin{cases} x^2+8x, & x \le -1 \\ x, & -1 < x \le 1 \\ -1, & x > 1 \end{cases}$

$f(-3)$. $x = -3 \le -1 \Rightarrow f(-3) = (-3)^2 + 8(-3) = 9 - 24 = -15$.

$f(-\frac{3}{2}) \Rightarrow x = -\frac{3}{2} \le -1 \Rightarrow f(-\frac{3}{2}) = (-\frac{3}{2})^2 + 8(-\frac{3}{2}) = \frac{9}{4} - \frac{24}{2} = \frac{9}{4} - \frac{48}{4} = -\frac{39}{4}$

$f(-1)$. $x = -1 \le -1 \Rightarrow f(-1) = (-1)^2 + 8(-1) = -7$; $f(0)$, $x = 0$, $-1 < 0 \le 1 \Rightarrow f(0) = 0$.

$f(45)$. $x = 45 > 1 \Rightarrow f(45) = -1$.

4. $f(x) = \frac{2}{x+7}$. $f(a) = \frac{2}{a+7}$; $f(a+h) = \frac{2}{a+h+7}$; $\frac{f(a+h) - f(a)}{h} = \frac{1}{h}\left(f(a+h) - f(a)\right) = \frac{1}{h}\left(\frac{2}{a+h+7} - \frac{2}{a+7}\right)$

$= \frac{1}{h}\left(\frac{2(a+7) - 2(a+h+7)}{(a+7)(a+h+7)}\right) = \frac{1}{h}\left(\frac{2a+14-2a-2h-14}{(a+h+7)(a+7)}\right) = \frac{1}{h}\left(\frac{-2h}{(a+h+7)(a+7)}\right)$

$= \frac{-2}{(a+h+7)(a+7)}$.

5. $f(x) = 5x^2+9$. Domain: $x \in \mathbb{R}$ – we can give any value we want to the function.
Range: $\{y \in \mathbb{R} \mid y \ge 9\} = [9, \infty)$.

6. $f(x) = 8x$, $-6 \le x \le 3$. Domain: Given to us! $-6 \le x \le 3$.
Range: Minimum: $f(-6) = 8(-6) = -48$; max: $f(3) = 8(3) = 24 \Rightarrow [-48, 24]$.

7. Domain $f(x) = \frac{x+5}{x^2-4}$. The problems would occur in the denominator: $x^2-4 = 0 \Rightarrow x = \pm 2$.
Thus, the denominator is 0 here, which is undefined, so we exclude them.
Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

8. Domain $f(t) = \sqrt{t+7}$. $\sqrt{t+7} \ge 0 \Rightarrow t+7 \ge 0 \Rightarrow t \ge -7$. Domain: $[-7, \infty)$

9. Domain: $f(x) = \sqrt{9-7x}$ $\sqrt{9-7x} \ge 0 \Rightarrow 9-7x \ge 0 \Rightarrow 9 \ge 7x \Rightarrow \frac{9}{7} \ge x \Rightarrow (-\infty, \frac{9}{7}]$.

10. $f(x) = \frac{(x+7)^2}{\sqrt{5x-1}}$. $\sqrt{5x-1} > 0 \Rightarrow 5x-1 > 0 \Rightarrow 5x > 1 \Rightarrow x > \frac{1}{5}$. Domain: $(\frac{1}{5}, \infty)$.
Note no equals!